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Mass bounds of the lightest CP-even Higgs boson in the two-Higgs-doublet model

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Abstract

The upper and the lower bounds of the lightest CP-even Higgs-boson mass (m_h) are discussed in the two-Higgs-doublet model (2HDM) with a softly-broken discrete symmetry. They are obtained as a function of a cut-off scale Λ ($\leq 10^{19}$ GeV) by imposing the conditions in which the running coupling constants neither blow up nor fall down below Λ. In comparison with the standard model (SM), although the upper bound does not change very much, the lower bound is considerably reduced. In the decoupling regime where only one Higgs boson (h) becomes much lighter than the others, the lower bound is given, for example, by about 100 GeV for $\Lambda = 10^{19}$ GeV and $m_t = 175$ GeV, which is smaller by about 40 GeV than the corresponding lower bound in the SM. In generic cases, m_h is no longer bounded from below by

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these conditions. If we consider the $b \to s\gamma$ constraint, small values of m_h are excluded in Model II of the 2HDM.

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After the discovery of the top quark, the Higgs sector is the last remaining part yet to be confirmed in the Standard Model (SM). Experimental efforts of *Higgs Hunting* are being at a climax in near future at LEP II, TeV 33 and LHC. Discovery of the Higgs particle is important not only in confirming the mechanism of the electroweak gauge-symmetry breaking but also in providing us useful information on physics beyond the SM. Although the mass of the Higgs boson is a free parameter in the minimal SM, we can obtain its mass bounds by imposing some theoretical assumptions. If we require the vacuum stability and the validity of perturbation theory below a given cut-off scale Λ, we can determine the lower and the upper bounds of the Higgs boson mass as a function of Λ, respectively. Allowed region of the Higgs boson and the top quark masses in the SM was examined in ref. [1]. For example, for the Plank scale $m_{Pl} \sim 10^{19}$ GeV as Λ, the lower and the upper bounds become about 145 and 175 GeV at $m_t = 175$ GeV, respectively. This has been reexamined by taking into account the two-loop beta function in ref. [2]. In the minimal supersymmetric standard model (MSSM), on the other hand, the theoretical upper bound on the lightest CP-even Higgs boson mass is given by about 120 GeV for $m_t = 175$ GeV and $m_{\rm stop} = 1$ TeV [3, 4, 5]. Also, in extended versions of the supersymmetric (SUSY) SM, we can obtain upper bounds, if we demand that all dimensionless coupling constants remain perturbative up to the GUT scale [6].

In this letter, we investigate the upper and the lower bounds of the lightest CP-even Higgs-boson mass in the two-Higgs-doublet model (2HDM) with a softly-breaking discrete symmetry by requiring the vacuum stability and the validity of perturbation theory. By a similar method as used in the SM, we can determine these mass bounds as a function of a cut-off scale Λ. In the 2HDM, a discrete symmetry is often assumed in order to suppress the flavor changing neutral current (FCNC) in a natural way [7]. According to the couplings with quarks, the 2HDM with such discrete symmetry is classified in two types; namely, one where only one Higgs doublet has Yukawa couplings with the quarks and leptons (Model I), and the other where the one Higgs doublet interacts only with the down-type quarks and leptons and the second one only with up-type quarks (Model II) [8]. In this letter, we also include soft-breaking terms of the discrete symmetry in the Higgs potential. Inclusion of these terms does not induce the FCNC problem and may

be necessary to avoid the domain wall problem [9]. There have been several works on the Higgs mass bounds in the 2HDM without the soft-breaking term [10, 11, 12, 13, 14]. Our analysis is a generalization of these works to the case with the soft-breaking terms. The results are qualitatively different from the previous works in the region of the large soft-breaking mass, where only one neutral Higgs boson becomes light and the others are much heavier and decouple from the electroweak scale. The lower bound of the lightest Higgs boson mass in this case is much reduced in comparison with that in the SM. For example for $\Lambda = 10^{19}$ GeV and $m_t = 175$ GeV, while the upper bound is about 175 GeV, which is the almost the same as in the SM, the lower bound is given by 100 GeV. This is considerably smaller than the similar lower bound in the SM which is 145 GeV. For the region of the small soft-breaking mass, the lower and upper bounds depend on the soft-breaking mass and there is no longer bounded from below in the case without the soft-breaking mass. In Model II 2HDM the constraint from $b \to s\gamma$ branching ratio excludes the small mass region of the neutral Higgs boson.

The Higgs potential of the 2HDM is given for both Model I and Model II as [8]

$$\begin{aligned} V_{\rm 2HDM} &= m_1^2 \left|\varphi_1\right|^2 + m_2^2 \left|\varphi_2\right|^2 - m_3^2 \left(\varphi_1^\dagger \varphi_2 + \varphi_2^\dagger \varphi_1\right) + \frac{\lambda_1}{2} \left|\varphi_1\right|^4 + \frac{\lambda_2}{2} \left|\varphi_2\right|^4 \\ &\quad + \lambda_3 \left|\varphi_1\right|^2 \left|\varphi_2\right|^2 + \lambda_4 \left|\varphi_1^\dagger \varphi_2\right|^2 + \frac{\lambda_5}{2} \left\{ \left(\varphi_1^\dagger \varphi_2\right)^2 + \left(\varphi_2^\dagger \varphi_1\right)^2 \right\}, \end{aligned} \tag{1}$$

where we include the soft-breaking terms for the discrete symmetry. For simplicity, we take all the self-coupling constants and the mass parameters in (1) to be real. In Model II φ_1 has couplings with down-type quarks and leptons and φ_2 has couplings with up-type quarks, and only φ_2 has couplings with fermions in Model I.

From the above Higgs potential (1), it is straightforward to derive masses of the Higgs bosons assuming that there is no CP nor charge violation at vacuum. Defining the ratio of two vacuum expectation values by $\tan\beta = \langle \varphi_2 \rangle / \langle \varphi_1 \rangle$, the masses of the charged Higgs bosons ($\chi^\pm$) and CP-odd Higgs boson (χ_2) are expressed as $m_{\chi^\pm}^2 = M^2 - (\lambda_4 + \lambda_5) v^2/2$ and $m_{\chi_2}^2 = M^2 - \lambda_5 v^2$, respectively, where $M = m_3/\sqrt{\cos\beta \sin\beta}$ and $v \sim 246$ GeV. The two CP-even Higgs boson masses are obtained by diagonalizing the 2×2 matrix, where each component is given by $M_{11}^2 = v^2 \left(\lambda_1 \cos^4\beta + \lambda_2 \sin^4\beta + \frac{\lambda}{2} \sin^2 2\beta\right)$, $M_{12}^2 = M_{21}^2 = v^2 \sin 2\beta \left(-\lambda_1 \cos^2\beta + \lambda_2 \sin^2\beta + \lambda \cos 2\beta\right)/2$ and $M_{22}^2 = v^2 \left(\lambda_1 + \lambda_2 - 2\lambda\right) \sin^2\beta \cos^2\beta +$

M^2 where $\lambda \equiv \lambda_3 + \lambda_4 + \lambda_5$. The mass of the lighter (heavier) CP-even Higgs boson h (H) is then given by $m_{h,H}^2 = \left\{M_{11}^2 + M_{22}^2 \mp \sqrt{(M_{11}^2 - M_{22}^2)^2 + 4M_{12}^4}\right\}/2$. For the case of $v^2 \ll M^2$, they can be expressed by

$$
\begin{aligned}
m_h^2 &= v^2\left(\lambda_1 \cos^4\beta + \lambda_2 \sin^4\beta + \frac{\lambda}{2}\sin^2 2\beta\right) + \mathcal{O}(\frac{v^4}{M^2}), &(2)\\
m_H^2 &= M^2 + v^2\left(\lambda_1 + \lambda_2 - 2\lambda\right)\sin^2\beta\cos^2\beta + \mathcal{O}(\frac{v^4}{M^2}). &(3)
\end{aligned}
$$

Notice that the free parameter M characterizes properties of the Higgs bosons in this model. In the case of $M^2 \gg \lambda_i v^2$, the masses of all the Higgs bosons but h become close to M. In this region, these heavy Higgs bosons decouple from the low-energy observable due to the decoupling theorem [15] and below the scale M the model is effectively regarded as the SM with one Higgs doublet. On the other hand, if $M^2 \sim \lambda_i v^2$, the masses are controlled by the self-coupling constants, and thus the heavy Higgs bosons do not decouple and the lightest CP-even Higgs boson can have a different property from the SM Higgs boson [16].

Let us discuss the conditions for validity of perturbation theory and the vacuum stability. For the first condition, we require that the running coupling constants of the Higgs self-couplings and the Yukawa couplings do not blow up below a certain energy scale Λ;

$$\forall\, \lambda_i(\mu) < 8\pi,\ y_t^2(\mu) < 4\pi\,, \tag{4}$$

for a renormalization scale μ less than Λ. For the requirement of the vacuum stability, we assume that the quartic interaction terms in the potential do not give negative contribution for all directions of scaler fields at each energy scale up to Λ. This condition leads to

$$
\begin{aligned}
&\lambda_1(\mu) > 0,\quad \lambda_2(\mu) > 0,\\
&\sqrt{\lambda_1(\mu)\lambda_2(\mu)} + \lambda_3(\mu) + \min\left[0, \lambda_4(\mu) + \lambda_5(\mu), \lambda_4(\mu) - \lambda_5(\mu)\right] > 0, \qquad (5)
\end{aligned}
$$

for $\mu < \Lambda$. We also require that the tree-level Higgs potential at the weak scale does not have any global minimum except for the one we consider. In particular, we assume that there is no CP nor charge breaking at the global minimum [4]. These conditions imposed

[4] The vacuum stability condition here is slightly different from that in ref. [14], where they have put $\lambda_4(\mu) + \lambda_5(\mu) < 0$ and $\lambda_5(\mu) < 0$ below $\mu < \Lambda$ in addition to (5) in the model with $M^2 = 0$. In the case of $M^2 \sim 0$, our condition is essentially the same as that in ref. [14], because we then have $\lambda_4 + \lambda_5 < 0$ and $\lambda_5 < 0$ at the electroweak scale from the positiveness of the squared-masses of $\chi^{\pm}$ and χ_2 and we can show that these inequalities tend to be preserved at higher energy scale according to the 2HDM RGE's.

on the coupling constants at a high energy region are transmitted into constraints on the coupling constants at the electroweak scale and then on the masses of Higgs bosons.

In the decoupling case where $M^2 \gg \lambda_i v^2$, there can be a sizable correction on the lightest CP-even Higgs boson mass at a low energy scale. In order to include this effect, instead of calculating the λ_i's at the weak scale from the RGE and using the tree-level mass formulas, we adopt the following procedure. We determine the λ_i at the scale M by using the 2HDM RGE in the region between Λ and M, and then calculate the CP-even Higgs boson mass according to the tree-level formulas. Since the effective theory below M is just the SM with one Higgs doublet, we use the SM RGE from M to m_h to evaluate the lightest Higgs boson mass. Although this procedure is not really justified for $M^2 \sim \lambda_i v^2$, we calculate the mass in this way because the correction from the SM RGE is numerically very small in such case.

In our analysis, we use the 1-loop RGE's for the SM and the 2HDM which are found, for example, in ref. [12, 17]. We only consider the top-Yukawa coupling contribution as the Yukawa interaction. The running top mass is defined as $\overline{m_t}(\mu) = \frac{1}{\sqrt{2}} y_t(\mu) v \sin\beta$ and it is related to the pole mass m_t by $\overline{m_t}(m_t) = m_t(1 - \frac{4}{3\pi}\alpha_s(m_t))$. There are important phenomenological constraints on the 2HDM. From the low-energy electroweak precision tests, the ρ parameter should be closed to unity, which means that the custodial $SU(2)_V$ symmetry should not be badly broken in the Higgs sector. We evaluate the 2HDM contribution to the ρ parameter according to refs. [18]. Taking account of the experimental data up to 95% CL [19], we here set the condition $\Delta\rho_{\text{2HDM}} = -0.0020 - 0.00049 \frac{m_t - 175\text{GeV}}{5\text{GeV}} \pm 0.0027$ for our analysis, where $\Delta\rho_{\text{2HDM}}$ is the extra contribution of the 2HDM to the ρ parameter[5]. Another experimental constraint is obtained from the $b \to s\gamma$ measurement [20]. It is known that there is very strong constraint on the charged-Higgs boson mass from this process in the case of Model II, while Model I is not strongly constrained. We calculate the $b \to s\gamma$ branching ratio with the next-to-leading order QCD correction [21] and use its constraint to determine the allowed region of the parameter space.

[5] We here set the reference value of the SM Higgs mass into 100 GeV. We also include uncertainties from the strong coupling constant and the electromagnetic coupling constant at the Z pole for our evaluation of the ρ paremeter.

In the actual analysis, we first fix parameter sets of m_h, $\tan\beta$ and M. Since the Higgs potential contains three masses and five coupling constants, the number of free parameters is four with fixing $v = 246$ GeV for each set of the parameter choice. We examine four-dimensional parameter space of λ_1, λ_2, λ_4 and λ_5 under the experimental constraints above and obtain a maximum scale Λ where one of the conditions (4) and (5) is broken. We also put $m_Z = 91.19$ GeV and $\alpha_S(m_Z) = 0.118$. The mass of the top quark is fixed as 175 GeV in our main analysis and later the dependence on m_t is discussed.

Let us first consider the case of the decoupling regime ($v^2 \ll M^2$). All the Higgs bosons but h are all heavy and their masses are almost degenerate around M. Fig. 1 shows that the contour plot of each Λ ($= 10^{19}, 10^{16}, 10^{13}, 10^{10}, 10^{7}, 10^{4}$ GeV) for $M = 1000$ GeV on the m_h-$\tan\beta$ plane. The $\tan\beta$ dependence is not so sensitive except for the small $\tan\beta$ region where the top-Yukawa coupling constant blows up at a low energy scale. For the smaller values of m_h, λ_2 tends to become negative because of the negative effect of y_t^4-term in the RGE for λ_2. On the other hand, for a large value of m_h, λ_2 blows up at a low energy scale. There is no difference between Model I and Model II in the decoupling regime, because the constraint from $b \to s\gamma$ is not important in this case.

The qualitative result may be understood by looking at the RGE's. From eq. (2), m_h^2 is approximately given by $\lambda_2 v^2$ for $tan\beta \gg 1$, and the RGE for λ_2 is given by

$$16\pi^2\mu\frac{d\lambda_2}{d\mu} = 12\lambda_2^2 - 3\lambda_2(3g^2 + g'^2) + \frac{3}{2}g^4 + \frac{3}{4}(g^2 + g'^2)^2 + 12\lambda_2 y_t^2 - 12y_t^4 + A, \quad (6)$$

where $A = 2\lambda_3^2 + 2(\lambda_3 + \lambda_4)^2 + 2\lambda_5^2 > 0$. When we fix the coupling normalization by $m_H^{SM} = \sqrt{\lambda_{SM}}v$, the SM RGE for λ_{SM} is obtained by substituting λ_{SM} and y_t^{SM} to λ_2 and y_t in eq. (6) and neglecting the A term in the RHS. Thus the difference is only in the existence of the positive term A in eq. (6). This term works to improve the stability of vacuum to some extent, and the lower bound is expected to be reduced in the 2HDM.

Next we see the case of the mixing regime ($M = 100$ GeV $\sim m_z$), where the heavy Higgs masses are realized only by the large λ_i's ($i = 1 - 5$) and their mixing. In this case, the data from the low energy experiment strongly constrain the model. The contour plots for each Λ on m_h-$\tan\beta$ plane in Model I and Model II are shown in figs. 2(a) and 2(b), respectively. We can see in figs. 2(a) and 2(b) that there is an allowed region for $\Lambda = 10^{19}$ GeV in Model I, while the largest Λ is less than 10^4 GeV in Model II because the $b \to s\gamma$

measurement gives a strong constraint for Model II 2HDM. Note that the allowed region in fig. 2(a) lies around $m_h \sim m_Z$ ($\sim M$) for large $\tan\beta$. This is because that, in the region of $M^2 < \lambda_2 v^2$, the mass of the lighter CP-even Higgs boson h comes from $M_{22} \sim M$ and the heavier Higgs boson H has the mass of $M_{11} \sim \sqrt{\lambda_2} v$. On the other hand, in the decoupling regime, the situation is reversed and the h boson has the mass of $M_{11} \sim \sqrt{\lambda_2} v$.

We repeated the above analysis for various values of M and obtained the upper and lower bounds of the lightest CP-even Higgs boson masses for various cut-off scales, which are shown in the contour plots in the m_h-M plane in figs. 3, (a) and (b) for Model I and II, respectively. In fig 3(a), the qualitative behavior of the allowed region is understood from the above argument on the mass matrix. For the region of $M^2 \ll \lambda_2 v^2$, the allowed region of m_h lies around $m_h \sim M$, and that becomes along $\sqrt{\lambda_2} v$ and no longer depends on M for $M^2 \gg \lambda_2 v^2$. Though there are the upper bounds of m_h for each Λ, m_h is not bounded from below by our condition. Our results at $M = 0$ are consistent to those in [14]. If we take account of the experimental result of $b \to s\gamma$, m_h is bounded from below in the case of Model II as seen in fig. 3(b) because the small M region ($M \lesssim 350$ GeV) necessarily corresponds to the light charged Higgs boson mass and is excluded by the $b \to s\gamma$ constraint[6].

Finally, we show the figure in which the results in the SM and the 2HDM (Model I and II) are combined on m_h-M plane (fig. 4). For a reference, the upper and lower bounds of the lightest CP-even Higgs mass in the MSSM are also given for the case that the stop mass is 1 TeV. These lines are calculated by a similar method described in ref. [4]: namely we use the SUSY relation for Higgs self-coupling constants at the 1 TeV scale and use the 2HDM RGE between 1 TeV and M, and the SM RGE between M and m_h scale. In this figure, M is the CP-odd Higgs boson mass in the case of the MSSM. It is easy to observe from this figure that the difference of the bounds among the SM, the 2HDM(I) and the 2HDM(II). We here choose, as an example, $\Lambda = 10^{19}$ GeV for the results in the SM and

[6] For the estimation of theoretical uncertainties we added in quadratures the errors form the various input parameters. If we use more conservative way to add theoretical uncertainties for the $b \to s\gamma$ evaluation, the bound on the charged Higgs boson or on the M in Model II becomes rather smaller[21]. The lower bound of m_h due to the $b \to s\gamma$ constraint is then reduced by a few GeV according to the change of the allowed region of M.

the 2HDM at $m_t = 175$ GeV. While the upper bounds in these models are all around 175 GeV, the lower bounds are completely different; about 145 GeV in the SM, about 100 GeV in the Model II and no bound in Model I.

In order to see the top quark mass dependence of the above results, we have repeated the analysis for $m_t = 170$ GeV and 180 GeV. It turns out that the lower bound has sizable dependence of the top mass whereas the upper bound does not change very much. For example, the lower line for $\Lambda = 10^{19}$ GeV in the 2HDM shown in fig. 4 shifts to lower (upper) by 9 GeV for $m_t = 170$ (180) GeV at $M = 1000$ GeV, but the corresponding shift for the upper line is about 3 (4) GeV. In Table 1, we list the m_t dependence of the lightest CP-even Higgs mass bounds for each value of Λ in the SM and the 2HDM for $M = 1000$ GeV and for $M = 200$ GeV (Model I).

We also comment on a question how much our results are improved if a higher order analysis is made in the effective potential method. In the SM, the next-to-leading order analysis of the effective potential shows that the lower bound reduces by about 10 GeV ($\Lambda = 10^{19}$ GeV) [2]. It may be then expected that a similar reduction of the lower bound would occur in the 2HDM by doing such higher order analysis.

We have analyzed the upper and the lower bounds of the lightest CP-even Higgs boson mass in the 2HDM with a softly-broken discrete symmetry by requiring that the running coupling constants neither blow up nor fall down below Λ. While the upper bound has been found to be almost the same as in SM, the lower bound turns out to be much reduced. In particular in the decoupling regime, both Model I and Model II give the lower bounds of about 100 GeV for $\Lambda = 10^{19}$ GeV, which is lower by 40 GeV than the SM result. In this regime, the properties of the lightest Higgs boson such as the production cross section and the decay branching ratios are almost the same as the SM Higgs boson. In this letter, we have not explicitly considered constraint from the Higgs boson search at LEP II, but if the Higgs boson is discovered with the mass around 100 GeV at LEP II or Tevatron experiment in near future and its property is quite similar to the SM Higgs boson, the 2HDM with very high cut-off scale is another candidate of models which predict such light Higgs boson along with the MSSM and its extensions.

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Table Caption

Table 1 A list of the lower and upper bounds of the lightest CP-even Higgs mass in GeV for each m_t $(= 170, 175, 180$ GeV$)$ and Λ $(= 10^{19}, 10^{16}, 10^{13}, 10^{10}, 10^{7}, 10^{4}$ GeV$)$ in the SM as well as the 2HDM for $M = 1000$ GeV and for $M = 200$ GeV (Model I). Model I and II give the same bounds for $M = 1000$ GeV.

Figure Captions

FIG. 1 The allowed region of the lightest CP even Higgs boson mass as a function of $\tan\beta$ for different values of the cut-off scale (Λ) for $M = 1000$ GeV in the 2HDM. The top mass is taken to be 175 GeV. For each Λ $(= 10^{19}, 10^{16}, 10^{13}, 10^{10}, 10^{7}, 10^{4}$ GeV$)$ the inside of the contour is allowed. There is no difference between Model I and Model II in this figure.

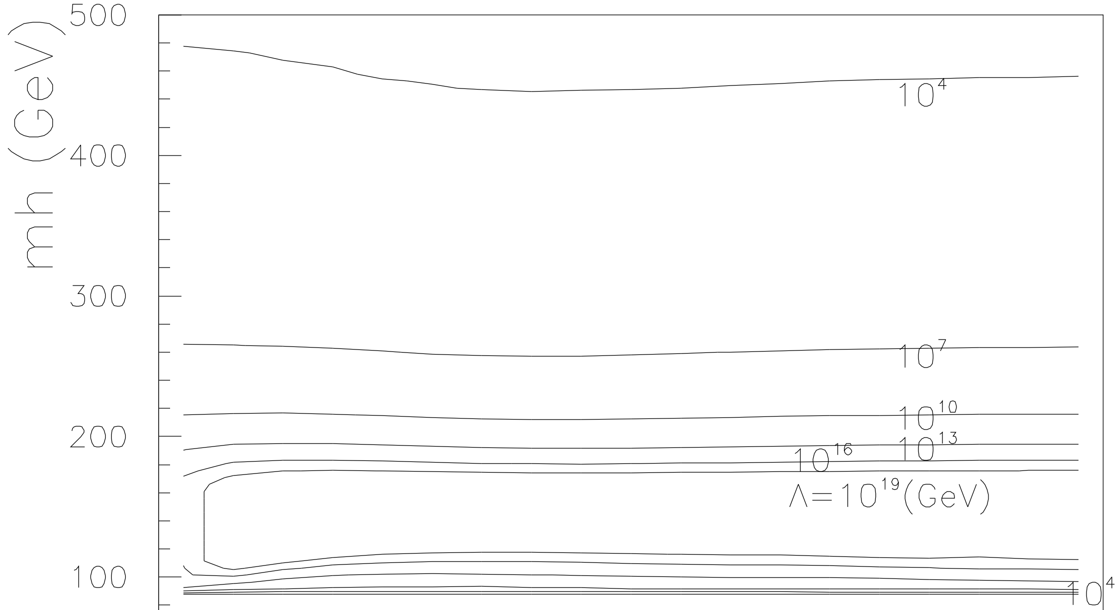
FIG. 2 The allowed region of the lightest CP even Higgs boson mass as a function of $\tan\beta$ for different values of Λ for $M = 100$ GeV in the Model I (a) and Model II (b) 2HDM. The top mass is taken to be 175 GeV. For the Model II lines for $\Lambda = 1000$ and 3000 GeV are shown.

FIG. 3 The upper and lower bounds of the lightest CP even Higgs boson mass as a function of M for different values of Λ in the Model I (a) and Model II (b) 2HDM for $m_t = 175$ GeV.

FIG. 4 The upper and the lower bounds of the lightest CP even Higgs boson mass in the Model I and II 2HDM and the SM Higgs boson mass for $\Lambda = 10^{19}$ GeV. The upper and lower bounds of the lightest CP even Higgs boson mass in the MSSM are also shown for the case that stop mass is 1 TeV. In this case M corresponds to the CP-odd Higgs boson mass in the MSSM.

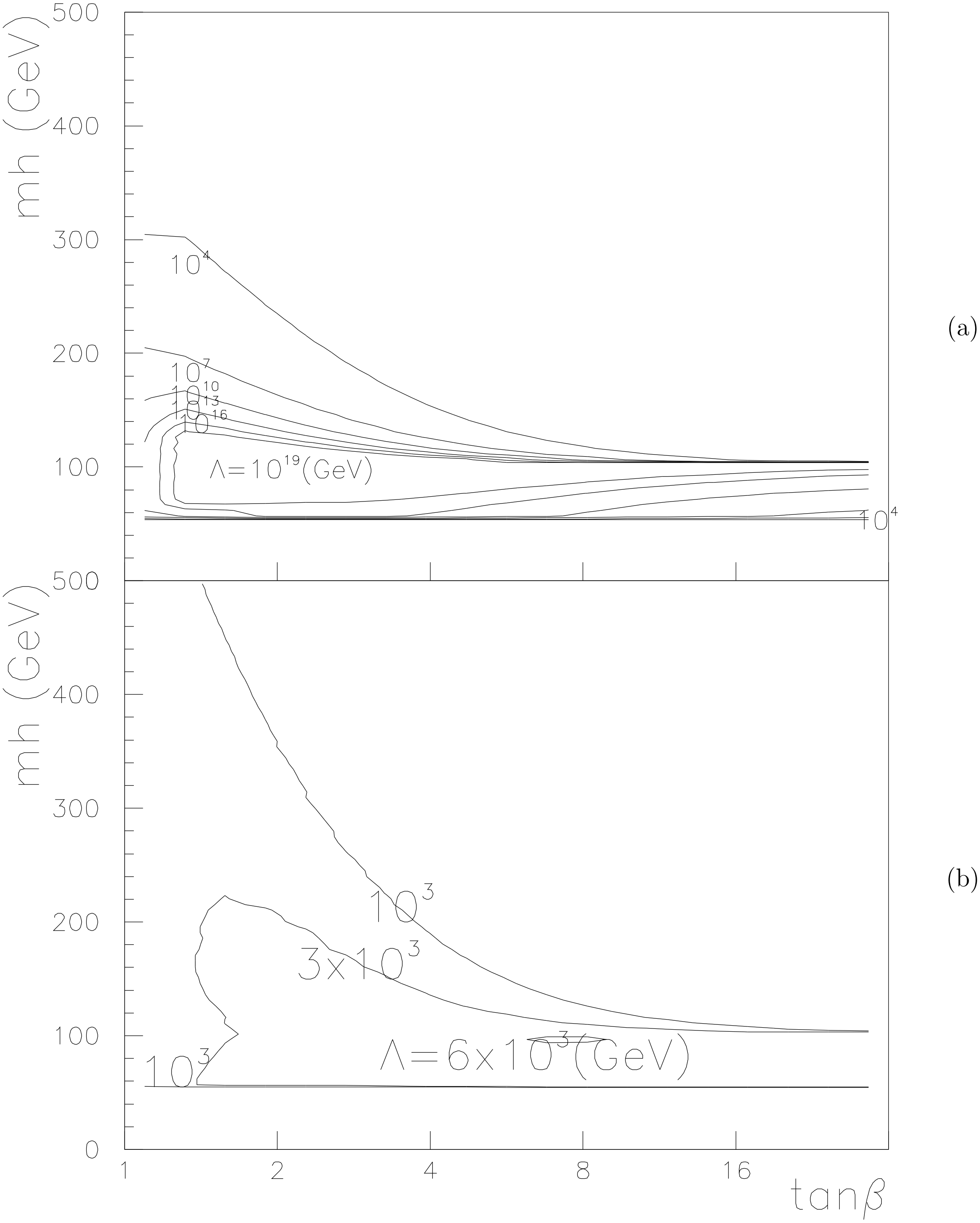
	Λ (GeV)	$m_t = 170$ GeV	$m_t = 175$ GeV	$m_t = 180$ GeV
Standard Model	10^{19}	133 - 172	143 - 175	153 - 179
2HDM ($M = 1000$GeV)		93 - 172	102 - 175	111 - 179
2HDM I ($M = 200$GeV)		79 - 171	84 - 175	91 - 179
	10^{16}	133 - 180	142 - 182	152 - 186
		89 - 180	96 - 183	104 - 186
		73 - 179	80 - 182	85 - 185
	10^{13}	132 - 192	141 - 194	150 - 197
		85 - 193	90 - 195	97 - 197
		68 - 191	72 - 193	77 - 195
	10^{10}	129 - 215	138 - 216	147 - 217
		85 - 216	89 - 216	93 - 218
		64 - 208	67 - 208	70 - 207
	10^{7}	122 - 264	130 - 264	138 - 264
		84 - 266	88 - 266	93 - 265
		64 - 238	67 - 241	69 - 241
	10^{4}	101 - 460	107 - 458	113 - 458
		84 - 480	88 - 480	92 - 478
		63 - 343	66 - 342	68 - 342

Table 1



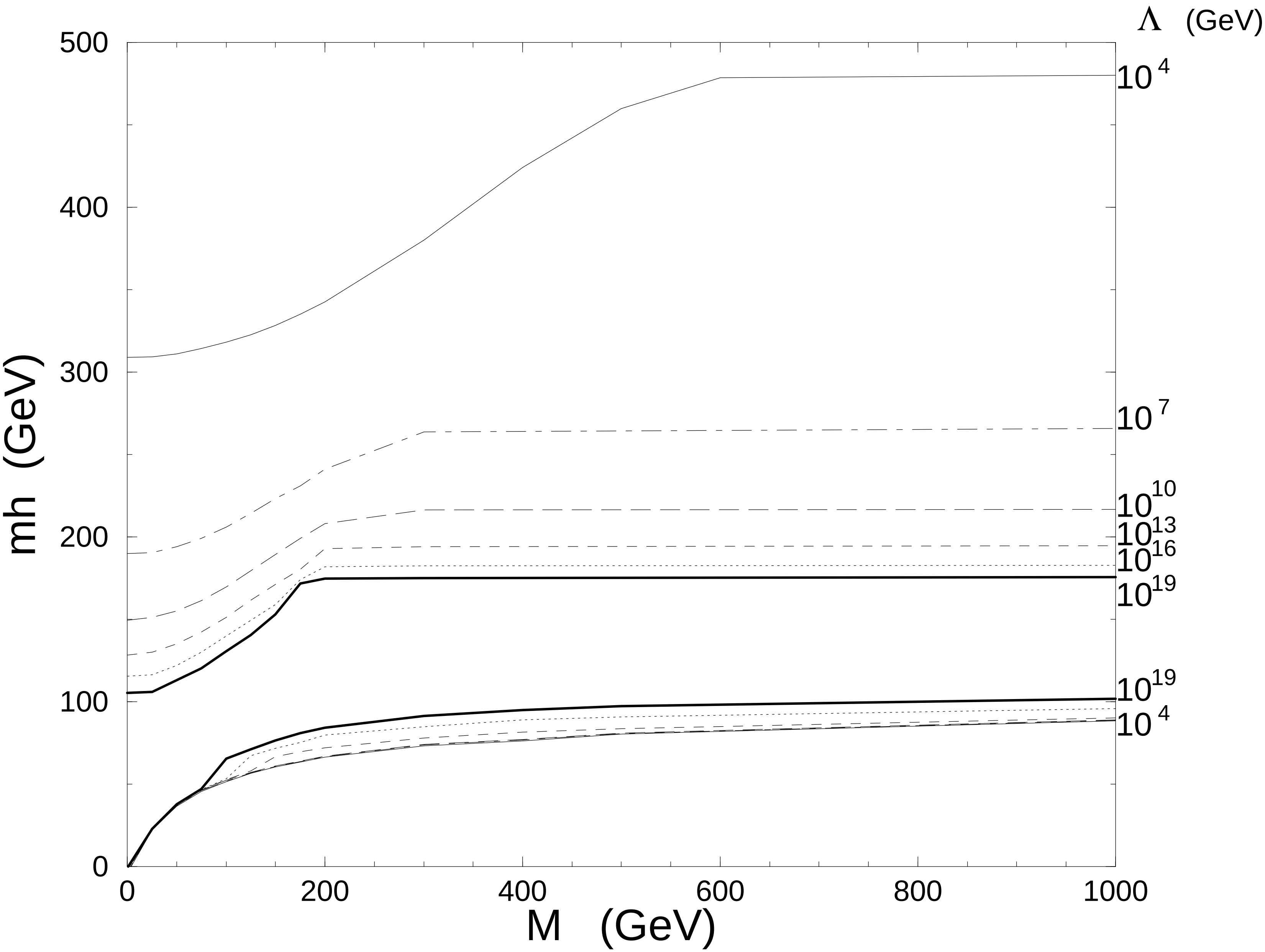
mh (GeV)
500
400
300
200
100
0
1
2
4
8
16
tanβ
10^4
10^7
10^{10}
10^{13}
10^{16}
$\Lambda=10^{19}$(GeV)
10^4

Fig. 1



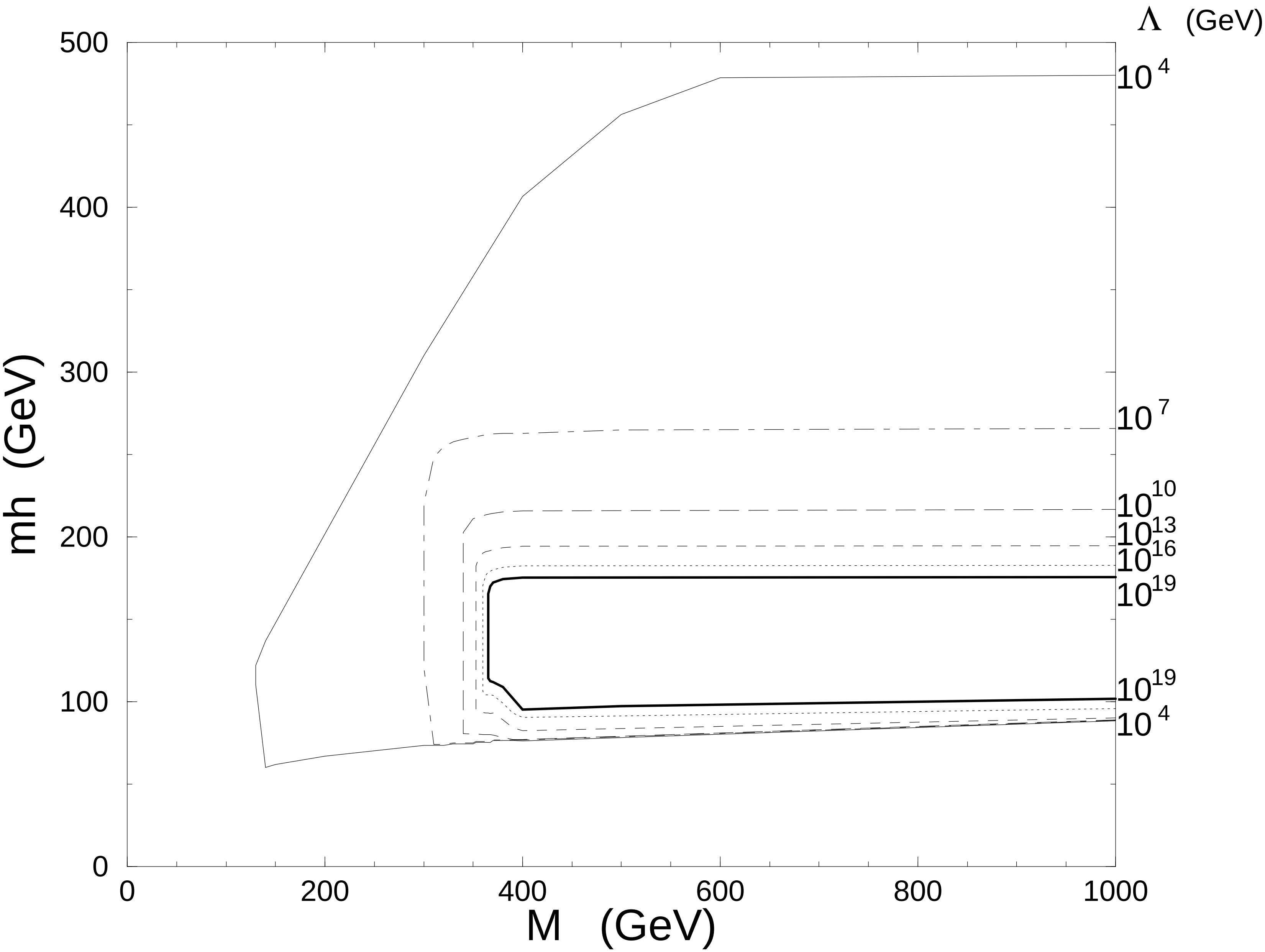
mh (GeV)
500
400
300
200
100
10^4
10^7
10^{10}
10^{13}
10^{16}
$\Lambda=10^{19}$(GeV)
10^4
(a)
mh (GeV)
500
400
300
200
100
0
10^3
3×10^3
10^3
$\Lambda=6\times10^3$(GeV)
(b)
1
2
4
8
16
tanβ

Fig. 2



Λ (GeV)
500
400
300
200
100
0
mh (GeV)
0
200
400
600
800
1000
M (GeV)
10^4
10^7
10^{10}
10^{13}
10^{16}
10^{19}
10^{19}
10^4

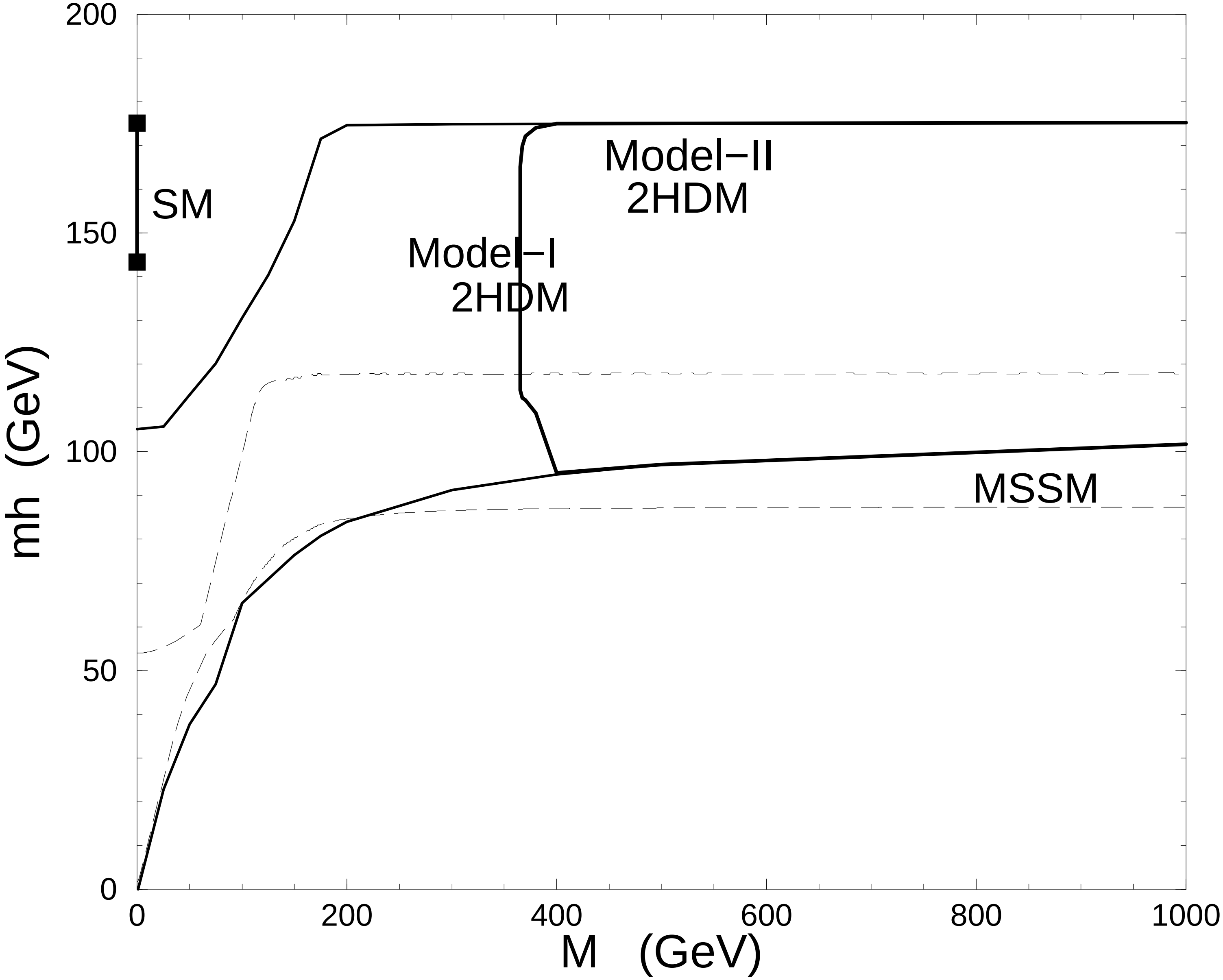
Fig. 3(a)



Λ (GeV)
500
400
300
200
100
0
mh (GeV)
0
200
400
600
800
1000
M (GeV)
10^4
10^7
10^{10}
10^{13}
10^{16}
10^{19}
10^{19}
10^4

Fig. 3(b)

Fig. 4



200
150
100
50
0
mh (GeV)
0
200
400
600
800
1000
M (GeV)
SM
Model−I
2HDM
Model−II
2HDM
MSSM